Energy Balance for Lumped Control Volume (Generally applicable for Biot # < 0.1)

Over a time interval Δt : $E_{in} - E_{out} + E_{gen} = \Delta E_{stored}$ (Joules)

Rate Equation at any instant : $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{stored}$ (Watts)

Surface Energy Balance : $\sum \dot{E}_{in} = \sum \dot{E}_{out}$ (Watts)

<u>Heat Diffusion Equations (differential elements)</u>

Cartesian coordinates: $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial t}$$

Cylindrical coordinates: $$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial t}$$

Spherical coordinates: $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\vartheta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial \vartheta}\left(k\sin\vartheta\frac{\partial T}{\partial \vartheta}\right)+\dot{q}=\rho c\frac{\partial T}{\partial t}$$

The heat flux vector from Fourier's Law is $\vec{\boldsymbol{q}}'' = -k\nabla T$

Cartesian	Cylindrical	Spherical
$\vec{\boldsymbol{q}} = -k\left(\frac{\partial T}{\partial x}\hat{\imath}+\frac{\partial T}{\partial y}\hat{\jmath}+\frac{\partial T}{\partial z}\hat{k}\right)$	$\vec{\boldsymbol{q}} = -k\left(\frac{\partial T}{\partial r}\hat{\imath}+\frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\jmath}+\frac{\partial T}{\partial z}\hat{k}\right)$	$\vec{\boldsymbol{q}} = -k\left(\frac{\partial T}{\partial r}\hat{\imath}+\frac{1}{r}\frac{\partial T}{\partial \vartheta}\hat{\jmath}+\frac{1}{r\sin\vartheta}\frac{\partial T}{\partial \phi}\hat{k}\right)$

Convection rate (Watts) from solid surface at T_s: $q = hA_s(T_s - T_\infty)$

Radiation from small gray surface of emissivity ε inside large enclosure at T_{surr}: $q = \varepsilon\sigma A_s(T^4 - T_{surr}^4)$

Resistance analog for steady conduction with no thermal generation : $q = \frac{\Delta T}{\text{Resistance}}$

Steady-state 2-D Conduction with no thermal generation and Shape Factor S : $q = S\,k\,\Delta T_{1-2}$

Biot number: $Bi = hL_C/k$ **Fourier Number**: $Fo = \alpha t/L_C^2$

Semi-Infinite solid solutions:

Constant surface temperature T_s :

$$\frac{T-T_s}{T_i-T_s}=\frac{2}{\sqrt{\pi}}\int_0^\eta \exp(-u^2)\,du \equiv erf\eta = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Where $\eta \equiv x/(4\alpha t)^{1/2}$

Constant Surface Heat Flux q_s'':

$$T(x,t)-T_i=\frac{2q_s''\sqrt{\alpha t/\pi}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right)-\frac{q_s''x}{k}\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Surface Convection:

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

1-D Transient Exact Solutions for Planar, Cylindrical, Spherical Systems with Convection Boundary Condition

Plane wall	Infinitely long cylinder	Sphere
$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\cos(\zeta_n x^*)$	$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\,\mathrm{J}_0(\zeta_n r^*)$	$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\frac{1}{\zeta_m r^*}\sin(\zeta_n r^*)$

Where $\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty}$